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ON THE FLOW AT THE REAR OF A TWO-DIMENSIONAL SUPERSONIC AIRFOIL WITH THICKNESS

by
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A treatment of the problem of the flow deflection immediately rearward of a two-dimensional supersonic airfoil was given by Kahane and Lees[1] in 1948. Their analysis was carried out on the basis of oblique shock and Prandtl-Meyer wave theory utilizing the fact that the dynamic and static pressure change through isentropic and shock waves can be expressed as a power series in the flow deflection angle. Since to the writer's knowledge this is the only place where the dynamic pressure coefficients have been published, it was thought worthwhile to point out that one of the coefficients in the expansion for an isentropic wave (also appearing in the shock expansion) appears to be incorrect.

The authors expressed the change in dynamic pressure through an isentropic wave as

$$\Delta q = q_1\left(\pm G_1\theta + G_2\theta^2 \pm G_3\theta^3 + \cdots\right)$$

and through a shock wave as

$$\Delta q = q_1\left(\pm G_1\theta + G_2\theta^2 \pm (G_3 - H)\theta^3 + \cdots\right)$$

where $q = \rho w^2/2$ (ρ = density, w = flow velocity), θ is the angle of deflection, positive in a counterclockwise sense, and q_1 is the dynamic pressure ahead of the wave. The corrected value for G_3 should be

$$G_3 = \frac{\frac{(3-\gamma)(2-\gamma)}{12}M^{10} + \frac{(-3\gamma^2+17\gamma-36)}{12}M^8 + \frac{(21-4\gamma)}{3}M^6 - 8M^4 + \frac{14}{3}M^2 - \frac{4}{3}}{(M^2-1)^{7/2}}$$

where M is the flow Mach number ahead of the wave.

Determination of the flow angle immediately rearward of a flat-plate airfoil at angle of attack, and for a thick airfoil at angle of attack where it was assumed that the configuration was such that four oblique shock waves were attached to the nose and tail of the airfoil, (see Fig. 1), was carried out. Fortunately the calculations did not involve the use of the incorrect coefficient G_3 directly. In spite of this however, for the thick airfoil shown in Fig. 1, the analytic

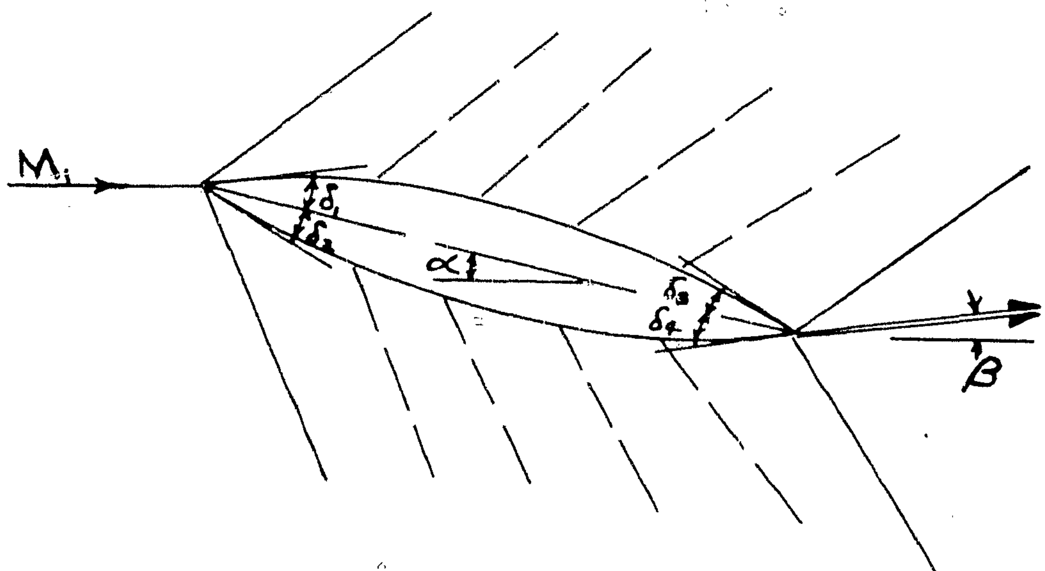

Fig. 1. The thick airfoil.

formulation of the flow deflection angle β to first order appears to be incorrect.

Recalculation of β gives

$$\beta = \frac{D}{2C_1}\left\{(\delta_3+\alpha)^3 + (\delta_1-\alpha)^3 - (\delta_4-\alpha)^3 - (\delta_2+\alpha)^3\right\}$$
$$+\frac{E}{2C_1}\left\{(\delta_3+\alpha)^4 + (\delta_1-\alpha)^4 - (\delta_4-\alpha)^4 - (\delta_2+\alpha)^4\right\}$$
$$+\frac{H}{2}\left\{(\delta_2+\alpha)^4 - (\delta_1-\alpha)^4\right\} - \frac{\left(\frac{dD}{d\theta} + DG_1\right)}{2C_1}\left\{(\delta_3+\alpha)^4 - (\delta_4-\alpha)^4\right\}$$

where D, C_1 and E are the coefficients of θ in the following expansion for the static pressure change through an oblique shock wave:

$$\Delta p = p_1\left(\pm C_1\theta + C_2\theta^2 \pm (C_3 - D)\theta^3 + (C_4 - E)\theta^4 \pm \cdots\right)$$

The required coefficients for the determination of β are functions only of the free stream Mach number M_1, and the ratio of specific heats γ. Their values (other than E) are given in Ref. 1, while (C_4-E) may be found in Ref. 2. The computation of C_4 and hence E is easily carried out by use of the results of Miles[3].

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